Cooperative Bandwidth Sharing for 5G Heterogeneous Network Using Game Theory

Shitong Yuan, Qilian Liang
Department of Electrical Engineering
University of Texas at Arlington
Arlington, TX-76019-0016
Email:shitong.yuan@mavs.uta.edu, liang@uta.edu

***Abstract*—In recent years, 4G LTE networks have been deployed by some operators and some of them also sell Femto Cell service in the market. As a kind of Heterogeneous network in cellular communication, lots of work have been published in last few years. However, considered in next generation cellular communication system, some challenging features have been planned like: A super-efficient and fast mobile network. But current system cannot satisfy users and operator. In the next generation wireless communication system, at least three layers of Heterogeneous Networks (HetNets) are required to provide high efficiency bandwidth usage and high speed data throughput. The problem is the users distribution and their quality of service (QoS) request are random, and the number of users may vary through the time. In order to deal with these problems, this paper builds two games models to optimize the resource allocation corresponding to different situations. Analyze the spectrum efficiency and made a comparison between two games. By playing those games, cells can serve more users inside one cell, all users are fair to share the bandwidth according to their request and location. The whole system becomes more flexible and performance has been enhanced.**

***Index Terms*—Heterogeneous Networks (HetNets), Pico Cell, Femto Cell, Cooperative Game, Backwards Induction Algorithm.**

I. Introduction

The network's capacity and throughput is increasing and spectrum efficiency are getting better with new technologies applied in practice. However, few challenges have been coming in recent years. Firstly, majority of cellular communication users all over the world are still using 2nd or 3rd generation networks. As huge amount of users have started to use 4th generation network service, a larger capacity improvement is needed due to the heavy data traffic. Secondly in some public areas, like airport, school or mall, there is a shortage in wireless resources due to a huge data requirement and interference. Finally it is a market trend that operators need a more efficient and cheaper network. The invention of small cell (like femtocell) based architecture in cellular communication, especially the multi-layers heterogeneous network (HetNets) is an effective solution to the above problems. Small cells have some advantages such as ease of deployment, cheap and can increase the frequency reuse rate.

The interference issue in LTE HetNets is a hot topic in recent years. Different from regular cellular network, HetNets have inter-layer interference. Few solutions have been proposed to this issue such as power control based interference coordination method, base station dynamically coverage range approach. However, for the challenge mentioned above, HetNets need a new method to improve frequency spectrum efficiency. In this research, we propose a new wireless resources allocation method in multi-layer HetNets based on game theory. Multiple parameters will be introduced to affect the strategies and decision and to find Nash Equilibrium points.

In 3GPP LTE-Advanced release 10 has defined a small cell-delay Node which is located in a macro cell. And Coordinated Multi Point (CoMP) operation is defined in release 11 which allows the set of TX-RX points used in CoMP to be either at different locations, or be co-sited but providing coverage in different sectors and they can also belong to the same or different eNBs. Some researches on heterogeneous networks (HetNets) in next generation cellular communication system have been proposed [1] [2]. The research work on HetNets mainly focuses on inter-cell interference, power management, and resources allocation issues. In [3], authors designed distributed utility functions minimum SINR for small cells. A resource management scheme is proposed in [4] [5] [6] [7] based on cognitive radio (CR) technology, which allows small cells to sense the usage of resource in the network. Some hybrid access of small cell method is proposed in [8] [9] which can reduce the co-channel interference with limited user connections.

There are multiple optimization methods for resource allocation. Game theory, as one of these methods, has been applied to this kind of problem for the few years. A basic view of cooperation games and its properties were provided in [13]. In [15], authors made a detail discussion on network and hierarchies with cooperative games. A dynamic spectrum sharing game was built and analyzed in [16]. A repeated sharing game for spectrum allocation was built in [12].

However, the theory and applications of HetNets are rare in next generation cellular communication system and is reflected in the following aspects:

1. Usually only two layer networks (macro and femto) are considered in HetNets research. In 5G, base stations are dense deployed. Femtocell and picocells are widely deployed

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by operators especially in densely populated public places.

2. Most access game models are based on non-cooperative or using non-cooperative methods. In game theory, all games must be have a solution that is Nash Equilibrium (NE). But NE is not always the optimal solution for them. In cellular HetNets, base stations are belong to same operator, cooperation is a feasible method to increase the network capacity.

The contributions of this paper include: a cooperative game model for three layer cellular HetNets, Optimization the utility function to maximize the bandwidth allocation surplus, Simulations are performed to verify the performance improvement.

The rest of this paper is organized as follows. Section II introduces the structure of the network then establish a bandwidth sharing game. Some properties are claimed and backwards induction algorithm was used to derive the surplus in the cooperation. Utility and benefit function are formulated in Section III. Section IV illustrates the system performance through simulation. Conclusions are drawn in the last Section.

II. Construction of Network Model

A. Network Structure and Scenarios

The cells in LTE standard are considered as a regular Base Station (BS), which cover a large area with radius up to 62miles (100Km). In some indoor areas where the outdoor signals have a small SNR, Pico cells are deployed. The Pico cells are used to increase the network capacity in some public places with densely wireless service requests. The femto cell, generally, is defined as a home hot spot, which only provides service to few devices. To achieve a larger capacity, in this paper we assume that Femto cells are always working in ”Open Access Mode”, which means that all users can connect to Femto cells.

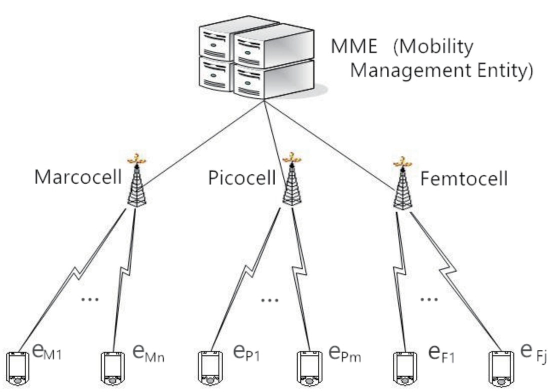

Fig. 1. Schematic Representation of HetNets

Fig. 1 summarizes the structure of HetNets which studied in this paper. Users are located in an indoor area will be and it is covered by macro cell, pico cell and femto cell. Three groups of users are attached to macro cell, pico cell and femto cell. And to simplify the game model, this paper does not consider the Coordinated Multi-points (CoMP) case but it is possible to deploy in LTE to get higher spectrum efficiency. In the CoMP case, users are allowed to connect with two cells simultaneously.However, in this paper, each user has a choice to connect to only one cell among the three to achieve the maximum utility. The utility here indicates allocated bandwidth or lower power consumption.

In real world, UEs cannot decide which cell to connect. All usersare managed by operator’s network such as Radio Access Network (RAN) in LTE. Based on the network capacity and reference signal receiving power (RSRP) reported by UE, eNodeB could require UE to perform multiple commands (handover, cell reselection, etc.). From operator’s point of view, more UEs should connect to femto cell or pico cell. Because it can release the load of macro cell which may have a huge number of users. Considering the above situations, this paper introduces a concept known as “User Stream”. Just like a river, the users flow from upstream to downstream. The upstream here can be considered as femto cell and the downstream is macro cell. In game theory, for the river sharing game, there are two doctrines restrict for different cells. The absolute territorial sovereignty (ATS) theory states that a country has absolute sovereignty over its territory for any river flowing through it. But the theory of unlimited territorial integrity (UTI) forbids a country to alter the natural conditions that may effect neighboring country on its own territory. Similarly, each cell can decide its subscriber set and how to allocate its resources. On the other hand, the cell can only “pick up” users uniformly or randomly, but not just leave the user who use heavy data service to downstream cells. One of this article’s important target is making a compromise between these two constraints.

Suppose there are totally X players (Users) in the network. All users have to choose a coalition i in the coalition set $I = \{M, P, F\}$, where “M”, “P”, “F” indicates macro cell, pico cell and femto cell, so that each user could maximize its utility. $M \in N, P \in N, F \in N$. As each user has to choose only one cell to attach, therefore $M \cap P = P \cap F = M \cap F = \emptyset$. Users and the cell constitute a coalition. Now this problem is divided into two parts:
1. Which cell should the user choose? It is typical coalition formation problem.
2. How the cell allocate the resources among its subscribers? It is a typical utility allocation problem.

B. The Game Model

Based on [11] [18], the characteristic function form of this game can be easily built.

Definition 1 (Bandwidth Sharing Game)

Considering a three layers HetNets, a cooperative bandwidth sharing game is defined as:

$$\langle I, S_n, U_n \rangle , \quad (1)$$

Where I is the player set, three cells are sharing the resources so $I = \{1, 2, 3\}$ and $n = \{1, ..., 2^N\}$ is the strategy vector index of player's strategy, X is the set of users. For example S_3 means the cell take strategy 3 and this vector indicates users in that cell should establish connection or not (0 means not). $S_{n,x} = (S_{1,1}, ..., S_{n,X}) \in \Re^X$. U is the utility set of players.

The utility of eNb $I \in \{1, 2, 3\}$ is composed by allocating w_i units of bandwidth to users and surplus from user.

$$U_i (w, e) = \sum_{i,x} [b_i (w_x) + Y(w_x - e_x)] , \quad (2)$$

where b_i is the benefit function which denotes the credit by providing service to users, $i \in 1, 2, 3$ Lets assume that at every $w_x > 0$, b_i is strictly concave and strictly increasing. If b_i' is the derivative of b_i, then $b_i' (w_x)$ tends to infinity as w_x goes to 0. That means each cell consume less wireless resources and provide better service to users getting better benefit or credit. $w_x \in W$ is the bandwidth allocation plan. e_x is the bandwidth that user actually used, Y is the function of surplus bandwidth.

The bandwidth in the paper is considered similar as goods, could be transferred to each other. Bandwidth allocated to each user w_x can only be used or occupied by subscribers of cell i. This makes our problem totally different from traditional economy allocation or sharing problems.

And the constraint must be added to the utility function:

$$\sum_{p=1}^{n} e_{1,p} + \sum_{d=1}^{m} e_{2,d} + \sum_{r=1}^{j} e_{3,r} \leq \sum_{x=1}^{X} W_x, \quad (3)$$

$$\sum_{x=1}^{X} (W_x - e_{i,x}) \geq 0 \quad (4)$$

Where $n + m + j = X$.

As equation (2) has a quasi-linear form, another important definition can be given based on its linear properties:

Definition 2 (Pareto Optimal Allocation)

Suppose $(w^*(N), e^*(N))$ is one of the allocation decisions made by Base Station, if and only if this allocation can maximize the utility function and does not waste any wireless resources (bandwidth), then this allocation is Pareto Optimal. Note that the Pareto optimal here means the system cannot improve any user service, and does not affect other users at the same time. $W'(N)$ can be called as an optimal occupancy plan.

Definition 3 (Marginal Benefits of The Game)

If the marginal benefits increase as one user connect to another cell, in another words, if two cells have different marginal benefits, some binding cooperative agreements or constraints must exist:

Suppose there are groups of users $U_P \in X, P = 1, ..., p$, and the corresponding benefits from these users can be presented as a list of positive values $\beta_{P\,P=1,...,p}$

if

$$b_i \left(W_{i,x}^* (U_p) \right) < b_{i'} \left(W_{i,x}^* (U_p) \right) , \quad (5)$$

$$\beta_{i,p} \geq \beta_{i',p} \quad (6)$$

then

$$b_{i'} \left(W_{i,x}^* (U_p) \right) = \beta_{i,p} \quad (7)$$

for every $i \in I$, $x \in U_p$ and every $p = 1, ..., P$.

And

$$\sum_{x \in U_P} \left(W_{i,x}^* (U_p) - e_{i,x} \right) = 0 \quad (8)$$

for every $p = 1, ..., P$.

C. Agreement of Transferable Surplus

The benefit function we derive in last section is a convex funcion. From the physical layer point of view, each cell has a capacity upper bound. The cell cannot serve too many users due to the system performance or channel limitation. On the other hand, if the marginal benefit of a user is higher for downstream cells, then the upstream cell can get extra credit for passing it to other cells. It is also possible that some of the passed users form Femto cell are connected to Pico cell but not Macro cell. Therefore, the utility of a cell (coalition) I depends on its own and other cells behavior.

Based on the above fact, lets assume Pico and Macro cell form a partition (not coalition) and both of them are trying to maximize their surplus for any amount of unused bandwidth by Femto cell (Pico cell). Besides, any amount of unallocated bandwidth can only be transferred to Pico cell or Macro cell, and each user that belongs to these coalitions is maximizing its surplus for its peers.

Similar to [18], we can easily describe players' and coalitions' relationship using mathematical language. Let P denote the partition of X and all users in that partition P are able to maximize their surplus of occupied bandwidth. Let $\bigcup_{K \in P} C(K) = K_1, ..., K_t$ and $K_1 < ... < K_t$. There is an algorithm called backwards induction (BIA) ([19][20][21]) which can find an optimal allocation plan for each unit of unallocated bandwidth received by the user. And let S' denote the surplus after allocation, $w^*(K_t, S')$ is the allocation plan for user K in algorithm's step t.

At step (t):

For all $S' \geq 0$, let $w^*(K_t, S')$ be the final optimal allocation plan for $\big(K_t, (e_{min(K_t)} + S', e_{K_t \min K_t}), b_{K_t}\big)$.

At step (t-1):

For all $S' \geq 0$, let $w^*(K_{t-1}, S')$ be the optimal allocation plan for $(K_{t-1}, (e_{minK_{t-1}} + S', e_{K_{t-1} \min K_{t-1}}), b_{K_{t-1}})$. Note that K_t and K_{t-1} have to be two different users in set P. After choose the plan $w^*(K_{t-1}, S')$, the surplus

$$S_t(w^*(K_{t-1}, S')) = S' + \sum_{x \in K_{t-1}} (e_x - W_x^*(K_{t-1}, S')) \quad (9)$$

is saved by K_{t-1} and give it to K_t. And K_t chooses the plan $w^*(K_t, S_t(w^*(K_{t-1}, S')))$.

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At step (j):

Let's define K_j as the bandwidth aggregated from previous users, and also $S' \geq 0$, $w(K_j, S')$ is an allocation (May Not be Optimal) for K_j if $S' + \sum(e_i - w_i(K_j, S') \geq 0$. Suppose for all users, $K_{j'}$ $(j' \in j+1, ..., t)$. Following the conclusion above, we can get:

$$S_{j+1}(w(K_j, S')) = S' + \sum_{x \in K_j} (e_x - W_x(K_j, S')) \quad (10)$$

be the bandwidth saved by K_j and passing to K_{j+1}. And also let

$$S_{j+2}(w(K_j, S')) = S_{j+2}(w^*(K_{j+1}, S_{j+1}(w(K_j, S')))) \quad (11)$$

be the bandwidth saved by K_j and K_{j+1} and passing to K_{j+2}. By deriving (11), we can easily get:

$$S_{j+k}(w(K_j, S')) = S_{j+k}(w^*(K_{j+K-1}, S_{j+k-1}(w(K_j, S')))) \quad (12)$$

which is presenting the bandwidth saved by $K_j, ..., K_{j+k-1}$ to K_{j+k}. Where $k \in 1, .., t-j$

Based on [18], if $K \in P$ and $K_j \subseteq K$, and if surplus $S' \geq 0$, then the allocation plan $w^*(K_j, S')$ solves

$$\max_{x(k_j, S')} \sum_{i \in k_j} b_i(w_i(k_j, S')) + \sum_{j' \in j+1, ..., t: K_{j'} \subseteq K} \sum_{i \in K_{j'}} b_i(w_i^*(k_{j'}, S_{j'}(w(K_j, S')))) \quad (13)$$

According to this equation, $w(k_j, S')$ maximizes K's surplus. Then we can conclude that the result of value of transferable surplus of this game can be calculated by BIA:

$$v = \sum_{i \in X} b_i(w^*(K_i, S_i(w^*(K_1, 0)))). \quad (14)$$

We will call v as the lower bound of coalition I given partition P, $P \in X$.

Definition 4 (Cooperative Lower Bound)

For coalitions I, let $v(I) = v(I, \{I, X/I\})$.
If for any partition $P \in X$, the cooperation does not make any positive effect on coalition I,
then

$$v(I) = v(I, P) \quad (15)$$

If for any partition $P \in X$, the cooperation does make some positive effects on coalition I,

$$v(I) \leq v(I, P) \quad (16)$$

We can further conclude that comparing to non-cooperation case, the cooperation does not decrease the benefit of a coalition which benefits greater than or equal to the lower bound.
Different from real upper bound, we can define an expectation upper bound for this game. This upper bound does not depend on how other coalitions (cells) behave, but is related with coalition $i's$ highest benefit and surplus it can achieve. Suppose other cells are absent for this game, coalition i is asked to choose a allocation plan $h(I) \in \Re_+^I$ to maximize $\sum_{i \in X} b_i(h_i(I))$ with following constraints:

$$\sum_{i \in P \cap I} h_i(I) \leq \sum_{i \in P} e_i \quad (17)$$

This maximization problem has only one solution due to its concave property. Suppose $h_i^*(I)$ is the solution, then the expected benefit of coalition I is:

$$S(I) = \sum_{i \in I} b_i(h_i^*(I)) \quad (18)$$

III. The Utility and Benefit Function

A utility function of bandwidth sharing game has been given in the last Section. Let's supplement more details to make it more straightforward in wireless communication problems. The utility function can be defined related to channel capacity. The first part of (2) which is benefit function denotes the capacity improvement by allocating bandwidth to users who subscribe to it; And the second part, which is quite important in cooperative game, shows its contribution to other players (cells). The surplus bandwidth from user can be used by other cell's subscribers.
The system makes a decision based users' RSRP. In [10], an equation on RSRP has been defined as:

$$RSRP_{i,x} = P_{MAX,i} \cdot L_{i,x}, \quad (19)$$

where $P_{MAX,i}$ is the maximum transmission power of eNB i and $L_{i,x}$ is the large scale channel gain including path loss, shadowing, penetration loss between eNB i and user x. In this paper, three layers HetNets are working in the same bandwidth, so there is strong interference between these cells. We assume the capacity of cell equals to 0 if there is no user subscribed to it. As the number of users goes on increasing (actually is the total bandwidth usage), the capacity goes on increasing. The capacity improvement by allocating bandwidth w_x to user $x \in X$ can be formulated as [17]:

$$C_{i,x} = w_x \cdot log_2(1 + \frac{p_x \cdot L_{i,x}}{\sigma_N^2 + P_{MAX,i' \in I, i \neq i'} \cdot L_{i',x}}), \quad (20)$$

However, the capacity improvement cannot simply be used as utility function. Because we built a cooperative model in last section. In other words, the cell should consider the bandwidth surplus after allocation. Let's assume users not only report their RSRP to the cell but also predict QoS. According to (13)(14), allocation plan $w(k_j, S')$ which considered surplus S' could maximize coalition surplus. A surplus factor S_f could be introduced to calculate the benefit function.

$$S_{f,x} = \max(C_{i,x}(\Delta L_{i,x}), \frac{q'}{q}) \quad (21)$$

Obviously, $S_{f,x} \in [0, 1]$ If $S_f = 0$, then user must have lost the connection, for example, power off the device. If $S_f = 1$, no change for the bandwidth usage. Based on this, we can derive our surplus function:

$$Y_i = \sum_x C_{i,x} \cdot \frac{1}{S_{f,x}}, \quad (22)$$

Then the utility function becomes:

$$U_i = \max(b_{i,x}(w_{i,x})) + \sum_x C_{i,x} \cdot \frac{1}{S_{f,x}}, \quad (23)$$

$$U = \arg\max_{w_x^*} \sum_{i=\{1,2,3\},x=1}^{X} C_{i,x} \quad (24)$$

IV. Performance Analysis

In the last section, we formulated the utility function and the surplus function. Consider the HetNets scenario in next generation cellular networks, we assume that macro cell cover a range of 600m which is smaller than the cell in LTE standard. And the pico cell's transmission range equals to 150m. Femto cell is an indoor BS that only covers a $25m \times 25m$ room. Suppose the system bandwidth is 20 MHz. The maximum transmission power for macro cell is 33 dBm, 20 dBm for picocell and 10 dBm for femto cell. Consider femto cell is available only in indoor environment, the number of users should not be large, so we assume there are up to 60 users in the system. When speaking of indoor environment, the wall penetration loss has to be considered, so we have set it as 20 dB.

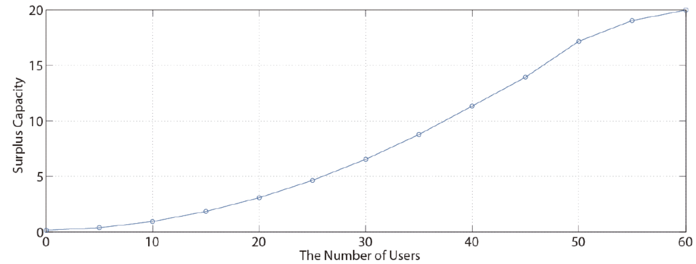

Fig. 2. Surplus Capacity versus number of users

Fig. 2. shows the sum of surplus as a function of the number of user. According to (13) of Section II, by applying backwards induction algorithm, the surplus is increasing as of users increasing. But it is a quite random value of each user's surplus because the behavior of user is uncertain. The result proves that the surplus function is monotonically increasing.
In our system, cells are sharing the bandwidth, as a consequence, interference may be a big issue for the sharing game. Consider this problem, we design a simulation related with interference. Suppose the distance between femto cell and pico cell equal to 100m.

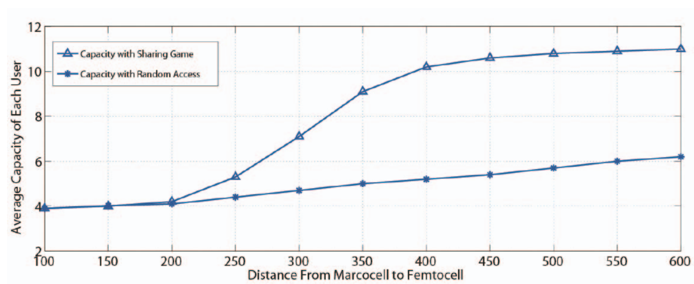

Fig. 3. Average capacity of each user as the distance between macro cell and femto cell increases

Fig. 3. shows average capacity of users versus the distance between femto cell and macro cell. It can be seen that the average capacity improve as distance increase. The main reason is that macro base station is transmitting with a high power. This signal is treated as noise at femto cell (as well as pico cell). Moreover, due to the cooperation between cells, if the channel condition is not good for a user, then its benefit and surplus value will be small. This directly leads the user to connect to a downstream cell to maximize its surplus. That's why the system is more efficient than the traditional access schemes which majorly depends on RSRP. A random access performance is shown in Fig. 3 to make a comparison to game theoretic method. The results turn out that there is a big plus on capacity by applying cooperative bandwidth sharing method.

V. Conclusion

In this article, a bandwidth sharing game has been proposed for 5G HetNets to optimize the wireless resources allocation by cooperation game among different cells. According to this model, cells are trying to maximize their surplus of allocation, while surplus could be reused by other cells' users. With the assumption of upstream base station, users are encouraged to connected to femto cell, which meets the operator's expectation. The utility function of this game was formulated with two parts: benefit function and surplus function. Both of them are based on information theory and indicates the capacity improvement of the system. Numerical results from simulation can be summarized as follows:

- Cooperative allocation get better performance with more subscribers.
- Spectrum efficiency is higher in a three layers HetNets compare to two layers only.
- The cooperative approach get better or equal performance than the non-cooperative in bandwidth sharing.

Some future works are possible based on this paper. If subscribers are allowed to connect with two cells, the problem will become a CoMP resources sharing problem. It is possible to get performance improvement if we consider the CoMP. On the other hand, the bandwidth allocation could be linked to carrier aggregation technology. With the cooperative game, spectrum efficiency may also improve in carrier aggregation.

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